Zoom Class 4/5/2020

Definite Integration

If $F'(x) = f(x)$, then $F(x)$ is called the primitive function of $f(x)$ and $\int f(x)\,dx = F(x) + C$, where 'C' is a constant of integration.

When $x = a$, $F(x) + C = F(a) + C$
When $x = b$ $F(x) + C = F(b) + C$

Now, the difference of the values of $F(x) + C$ at $x = b$ and $x = a$ is

$$= \{F(b) + C\} - \{F(a) + C\}$$
$$= \{F(b) - F(a)\}.$$

which is called the definite

integral of $f(x)$ between the limits 'a' and 'b'. In notation form it is expressed as

$$\int_a^b f(x)\,dx.$$

$$\therefore \boxed{\int_a^b f(x)\,dx = \Big[F(x)\Big]_a^b = F(b) - F(a)}$$

where $\int f(x)\,dx = F(x)$

Ques) Evaluate $\int_1^2 \left(x^2 - \frac{1}{x} + 1\right) dx$

Soln $I = \int_1^2 \left(x^2 - \frac{1}{x} + 1\right) dx = \left[\frac{x^3}{3} - \log x + x\right]_1^2$

$$= \left[\frac{2^3}{3} - \log 2 + 2\right] - \left[\frac{1^3}{3} - \log 1 + 1\right]$$

$= \frac{8}{3} - \frac{1}{3} - \{\log 2 + \log 1\} + 2 - 1$

$= \frac{7}{3} + 1 - \log 2 = \frac{10}{3} - \log 2$

Ans

Some properties of Definite Integrals

(i) $\int_a^b f(x)\,dx = \int_a^b f(z)\,dz$

where $\phi(x) = z$
and when $x = a$, $z = \phi(a)$
when $x = b$, $z = \phi(b)$

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(ii) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

(iii) $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

where $a < c < b$

(iv) $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$.

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(v) $\int_{-a}^a f(x)\,dx = 2\int_0^a f(x)\,dx)$, when $f(x)$ is an even function
$= 0$, when $f(x)$ is an odd function

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Proof of properties

(i) If $\int f(x)\,dx = \cancel{B}F(x)$

and $\int f(z)\,dz = F(z)$

$\therefore \int_a^b f(x)\,dx = \left[F(x)\right]_a^b = F(b) - F(a)$

$\int_a^b f(z)\,dz = \left[F(z)\right]_a^b = F(b) - F(a)$

$\therefore \int_a^b f(x)\,dx = \int_a^b f(z)\,dz$

(ii) $\int_a^b f(x)dx = \left[F(x)\right]_a^b = F(b) - F(a)$

$= -\{F(a) - F(b)\}$

$= -\int_b^a f(x)\,dx.$

(iii) $\int_a^b f(x)\,dx = \left[F(x)\right]_a^b$

$= F(b) - F(a)$

$= \{F(c) - F(a)\} + \{F(b) - F(c)\}$

$= \left[F(x)\right]_a^c + \left[F(x)\right]_c^b$

$= \int_a^c f(x)\,dx + \int_c^b f(x)\,dx.$

(iv) $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

Let $a-x = y \Rightarrow -dx = dy$

Also when $x = a$, $y = 0$
and when $x = 0$, $y = a$

$\therefore$ RHS $= \int_0^a f(a-x)dx$

$= \int_a^0 f(y)(-1)dy = -\int_a^0 f(y)dy$

$= \int_0^a f(y)dy = \int_0^a f(x)dx$ —— using(ii)

$=$ LHS.

Proved

(v) We know $\int_{-a}^a f(x)dx = \int_{-a}^0 f(x)dx + \int_0^a f(x)dx$ — (A)

Now for $\int_{-a}^0 f(x)dx$, Putting $x = -z$
$\Rightarrow dx = -dz$

and when $x = 0 \Rightarrow z = 0$
when $x = -a, \Rightarrow z = a$

$\Rightarrow \int_{-a}^0 f(x)dx = \int_a^0 f(-z)(-dz) = -\int_a^0 f(z)$ when $f(z) = f(-z)$

$= +\int_0^a f(z)dz = +\int_0^a f(x)dx$ — (B)

Using (B) in (A) we get

$$\int_{-a}^{a} f(x)\,dx = +\int_{0}^{a} f(x)\,dx + \int_{0}^{a} f(x)\,dx$$ ~~when~~

$$= 2\int_{0}^{a} f(x)\,dx.$$ when $f(x)$ is an ~~odd~~ even function.

and $$\int_{-a}^{0} f(x)\,dx = \int_{0}^{a} f(-z)(-dz)$$

$$= \int_{a}^{0} f(z)\,dz \rightarrow \text{when } f(-z) = -f(z)$$

$$= -\int_{0}^{a} f(z)\,dz = -\int_{0}^{a} f(x)\,dx \quad \text{— (C)}$$

Using (C) in (A) we get

$$\int_{-a}^{a} f(x)\,dx = -\int_{0}^{a} f(x)\,dx + \int_{0}^{a} f(x)\,dx$$

$$= 0$$ when $f(x)$ is an odd function.